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On the Equivalence Between Canonical Forms of Recursive Systematic Convolutional Transducers Based on Single Shift Registers

HORIA BALTA[1,2], CATHERINE DOUILLARD[3], (Senior Member, IEEE), AND ALEXANDRU ISAR[1], (Senior Member, IEEE)
[1]University Politehnica Timişoara, Timişoara 300223, Romania
[2]Valahia University of Târgovişte, Târgovişte 130024, Romania
[3]Institut Mines-Telecom/Telecom Bretagne, Paris Cedex 13 75634, France
Corresponding author: H. Balta (horia.balta@upt.ro)

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ABSTRACT Standardized turbo codes (TCs) use recursive systematic convolutional transducers of rate $b/(b+d)$, having a single feedback polynomial ($b+d$RSCT). In this paper, we investigate the realizability of the $b+d$RSCT set through two single shift register canonical forms (SSRCFs), called, in the theory of linear systems, constructibility, and controllability. The two investigated SSRCF are the adaptations, for the implementation of $b+d$RSCT, of the better-known canonical forms controller (constructibility) and observer (controllability). Constructibility is the implementation form actually used for convolutional transducers in TCs. This paper shows that any $b+1$RSCT can be implemented in a unique SSRCF observer. As a result, we build a function, $\xi:\mathcal{H}\rightarrow\mathcal{G}$, which has as definition domain the set of encoders in SSRCF constructibility, denoted by $\mathcal{H}$, and as codomain a subset of encoders in SSRCF observer, denoted by $\mathcal{G}$. By proving the noninjectivity and nonsurjectivity properties of the function ξ, we prove that $\mathcal{H}$ is redundant and incomplete in comparison with $\mathcal{G}$, i.e., the SSRCF observer is more efficient than the SSRCF constructibility for the implementation of $b+1$RSCT. We show that the redundancy of the set $\mathcal{H}$ is dependent on the memory m and on the number of inputs b of the considered $b+1$RSCT. In addition, the difference between $\mathcal{G}$ and $\xi(\mathcal{H})$ contains encoders with very good performance, when used in a TC structure. This difference is consistent for $m \approx b > 1$. The results on the realizability of the $b+1$RSCT allowed us some considerations on $b+d$RSCT, with $b, d > 1$, as well, for which we proposed the SSRCF controllability. These results could be useful in the design of TC based on exhaustive search. So, the proposed implementation form permits the design of new TCs, which cannot be conceived based on the actual form. It is possible, even probable, among these new TCs to find better performance than in the current communication standards, such as LTE, DVB, or deep-space communications.

INDEX TERMS Canonical forms, convolutional encoders, equivalence, encoding matrix, turbo codes.

I. INTRODUCTION

Convolutional codes (CCs) were introduced by P. Elias in 1955 [1]. Elias was referring exclusively to non-recursive and systematic CEs. In addition, in [1]-Fig. 5, Elias suggested a form of implementation (encoder) for non-recursive and systematic CCs comprising of a shift register, an adder and a mixer. In [1], Elias also introduced the list decoding algorithm for block codes, an algorithm called by Anderson [2], [3], the "M-algorithm".

CCs have been extensively studied since then. Wozencraft [2] proposed the first sequential decoding algorithm. A simpler and more efficient sequential decoding algorithm was proposed by Fano [4] in 1963. Also in 1963, Massey [5] proposed the threshold decoding. Massey was also one of the first to study the CCs' structural properties. Along with Sain [6], he defined the equivalence between two convolutional generator matrices (if they encode the same code). They showed that any CC can be encoded by a polynomial generating matrix. They also [7], [8] studied the existence conditions of a polynomial right inverse for a convolutional generator matrix. Costello showed [9] that any convolutional generator matrix is equivalent to o rational systematic encoding matrix. Massey and Costello proposed in [10] the use of non-systematic CEs with a